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## On the role of the commutator algebra for nonlinear supersymmetry

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### Abstract

We discuss a closure of commutator algebras for general functionals in terms of Nambu-Goldstone fermions and their derivative terms under nonlinear supersymmetry (NLSUSY) both in flat spacetime and in curved spacetime. We point out that variations of functionals for vector supermultiplets (uniquely) determine general LSUSY transformations for linear vector supermultiplets with general auxiliary fields in extended SUSY, where the closure of the commutator algebras for NLSUSY plays a crucial role.

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Supersymmetry (SUSY) is realized linearly and nonlinearly. Linear supersymmetric (LSUSY) theories in flat spacetime [1, 2] are given from the scalar and vector supermultiplets and their actions, while the Volkov-Akulov (VA) nonlinear supersymmetric (NLSUSY) theory [3] is expressed only in terms of Nambu-Goldstone (NG) fermions.

The LSUSY gives plain physical pictures but the meaning of the SUSY breaking is unclear. On the other hand, the NLSUSY gives less physical pictures but gives the robust SUSY breaking. The LSUSY and NLSUSY theories are related to each other and their relations are shown explicitly in  $N = 1$  and  $N = 2$  SUSY models by means of the linearization of NLSUSY [4]-[8]. In the relation between linear and nonlinear realizations of SUSY (NL/LSUSY relation) in flat spacetime, component fields of supermultiplets are expressed as functionals (composites) in terms of the NG fermions, which reproduce LSUSY transformations in LSUSY multiplets from the variations of those functionals under NLSUSY transformations for the NG fermions, and LSUSY actions are related to a NLSUSY one.

On the other hand, a (global) NLSUSY transformations for the NG fermions with a vierbein field is generalized to curved spacetime and an Einstein-Hilbert-type NLSUSY invariant action are constructed in NLSUSY general relativity (GR) [9, 10]. The NLSUSY-GR action contains the VA NLSUSY action in the cosmological term and its implications for the low energy physics is extracted from the NLSUSY action for  $N \geq 2$  SUSY. Therefore, in order to discuss the physical consequences of NLSUSY GR, it is an important and interesting problem to know through the NL/LSUSY relation the general structure of linear supermultiplets in extended SUSY which includes *general features of auxiliary fields* prior to adopting a specific gauge *a la* the Wess-Zumino gauge.

Towards understanding the general NL/LSUSY relation for  $N \geq 2$  SUSY theories and obtaining a general LSUSY supermultiplet which contributes to the linearized action of NLSUSY-GR theory, we discuss in this letter commutator algebras for general functionals in terms of the NG fermions under the NLSUSY transformations both in flat spacetime and in curved spacetime. In flat spacetime, we explicitly show that the commutator algebra for all Lorentz-tensor functionals of NG fermions and their first- and higher-order derivatives are closed. In curved spacetime, we also show a closure of the commutator algebra for all Lorentz- and Riemann-tensor functionals of the NG fermions, the vierbein and their first-order derivatives. Based on these arguments, we point out that variations of functionals for vector supermultiplets (uniquely) determine general LSUSY transformations for linear vector

supermultiplets in extended SUSY containing general auxiliary fields.

Let us briefly review the VA NLSUSY model [3] in extended SUSY. The fundamental action in terms of (Majorana) NG fermions  $\psi^i$ <sup>‡</sup> is written as

$$S_{\text{NLSUSY}} = -\frac{1}{2\kappa^2} \int d^4x |w|, \quad (1)$$

where  $\kappa$  is a dimensional constant whose dimension is  $(\text{mass})^{-2}$  and the determinant  $|w|$  is defined as

$$|w| = \det(w^a_b) = \det(\delta^a_b + t^a_b) \quad (2)$$

with  $t^a_b = -i\kappa^2 \bar{\psi}^i \gamma^a \partial_b \psi^i$ . The  $N$  NLSUSY action (1) is invariant under NLSUSY transformations of  $\psi^i$ ,

$$\delta_\zeta \psi^i = \frac{1}{\kappa} \zeta^i - i\kappa \bar{\zeta}^j \gamma^a \psi^j \partial_a \psi^i, \quad (3)$$

which are parametrized by means of constant (Majorana) spinor parameters  $\zeta^i$ . The NLSUSY transformations (3) satisfy a closed commutator algebra,

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_P(\Xi^a), \quad (4)$$

where  $\delta_P(\Xi^a)$  means a translation with the parameters  $\Xi^a = 2i\bar{\zeta}_1^i \gamma^a \zeta_2^i$ .

Here we consider Lorentz-tensor functionals in terms of  $\psi^i$  and its first- and higher-order derivatives  $(\partial\psi^i, \partial^2\psi^i, \dots, \partial^n\psi^i)$ ,

$$f_A^I = f_A^I(\psi^i, \partial\psi^i, \partial^2\psi^i, \dots, \partial^n\psi^i), \quad g_B^J = g_B^J(\psi^i, \partial\psi^i, \partial^2\psi^i, \dots, \partial^n\psi^i) \quad (5)$$

where the Lorentz indices  $A, B$  mean  $a, ab, \dots$ , etc. and the internal indices  $I, J$  are  $i, ij, \dots$ , etc. If we assume that they satisfy the commutator algebra (4) as

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] f_A^I = \Xi^a \partial_a f_A^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] g_B^J = \Xi^a \partial_a g_B^J. \quad (6)$$

then, we can show that the commutator algebra for their products  $f_A^I g_B^J$  are also closed as

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] (f_A^I g_B^J) = \Xi^a \partial_a (f_A^I g_B^J). \quad (7)$$

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<sup>‡</sup>The index  $i, j, \dots$  runs from 1 to  $N$  and Minkowski spacetime indices are denoted by  $a, b, \dots = 0, 1, 2, 3$ . The Minkowski spacetime metric is  $\eta^{ab} = \text{diag}(+, -, -, -) = \frac{1}{2}\{\gamma^a, \gamma^b\}$ . Riemann spacetime indices  $\mu, \nu, \dots = 0, 1, 2, 3$  are used in arguments for curved spacetime.

This means that the commutator algebra for all Lorentz-tensor functionals expressed by means of Eq.(5) are closed same as Eq.(4), since each field of  $(\psi^i, \partial\psi^i, \partial^2\psi^i, \dots, \partial^n\psi^i)$  satisfies the algebra (4), respectively.

Based on these discussions for the closure of the commutator algebra under the NLSUSY transformations (3), let us examine the variations of functionals  $f_A^I(\psi^i, |w|)$  which correspond to those of a  $N = 2$  vector supermultiplet in  $d = 2$  [11, 12]. In order to realize the commutator algebra (4), the two supertransformations of Eq.(5) have a general form,

$$\delta_{\zeta_1} \delta_{\zeta_2} f_A^I = \frac{1}{2} \Xi^a \partial_a f_A^I + \bar{\zeta}_1^k \gamma^B \zeta_2^l \partial^2 h_{AB}^{Ikl}, \quad (8)$$

where  $h_{AB}^{Ikl} = h_{AB}^{Ikl}(\psi^i, |w|)$  are functionals factorized with respect to  $\bar{\psi}^k \gamma_B \psi^l$ . Since the second terms  $\bar{\zeta}_1^k \gamma^B \zeta_2^l h_{AB}^{Ikl}$  are symmetric for exchanging the indices 1 and 2, they vanish in the commutator algebra (6), respectively. On the other hand, let us write the variations of  $f_A^I$  in a simple form as

$$\delta_\zeta f_A^I = i \not{\partial} b_{AC}^{Ik} \gamma^C \zeta^k, \quad (9)$$

by using functionals  $b_{AC}^{Ik} = b_{AC}^{Ik}(\psi^i, |w|)$ . In Eq.(9), when the  $f_A^I$  are fermionic functionals, the  $b_{AC}^{Ik}$  are bosonic ones and vice versa. Then, comparing Eq.(8) and Eq.(9) determines general LSUSY transformations of  $b_{AC}^{Ik}$  by regarding  $(f_A^I, h_{AB}^{Ikl}, b_{AC}^{Ik})$  as fermionic or bosonic components, e.g. as follows;

$$\delta_\zeta b_{AC}^{Ik} = \gamma_C f_A^I \zeta^k + i \not{\partial} h_{AB}^{Ikl} \gamma_C^B \zeta^l. \quad (10)$$

Therefore, we point out through the above heuristic arguments in extended SUSY that general LSUSY transformations for linear supermultiplets with general auxiliary fields are determined by means of the functionals  $f_A^I$ ,  $h_{AB}^{Ikl}$  and  $b_{AC}^{Ik}$  as a result of the closure of commutator algebra (4).

As a simple example of the above commutator-based LSUSY transformations of the composite fields, we can demonstrate the NL/LSUSY relation for the  $d = 2$ ,  $N = 2$  vector supermultiplet [11, 12].

The  $N = 2$  LSUSY invariant (free) action with a  $D$  term is written as

$$S_{N=2 \text{ gauge}} = \int d^2x \left\{ -\frac{1}{4} (F_{0ab})^2 + \frac{i}{2} \bar{\lambda}_0^i \not{\partial} \lambda_0^i + \frac{1}{2} (\partial_a A_0)^2 + \frac{1}{2} (\partial_a \phi_0)^2 + \frac{1}{2} D_0^2 - \frac{\xi}{\kappa} D_0 \right\}, \quad (11)$$

where  $(v_{0a}, \lambda_0^i, A_0, \phi_0, D_0)$  are components of the  $N = 2$  minimal (Wess-Zumino) gauge supermultiplet defined by using auxiliary fields  $(D, \Lambda^i, C)$  and the recombination of the components of the larger vector supermultiplet as follows;

$$(v_{0a}, \lambda_0^i, A_0, \phi_0, D_0) = (v_a, \lambda^i + i\partial\Lambda^i, M^{ii}, \phi, D + \square C). \quad (12)$$

The action (11) is invariant under LSUSY transformations for the most general components of the  $N = 2$  vector supermultiplet in Eq.(12), which satisfy the algebra (4), as follows;

$$\begin{aligned} \delta_\zeta \lambda^i &= D\zeta^i - i\partial M^{ij}\zeta^j - \frac{i}{2}\epsilon^{ij}\gamma_5\partial\phi\zeta^j - \frac{1}{2}\epsilon^{ij}\gamma^a\partial v_a\zeta^j, \\ \delta_\zeta D &= -i\bar{\zeta}^i\partial\lambda^i, \\ \delta_\zeta M^{11} &= \bar{\zeta}^1\lambda^1 + i\bar{\zeta}^2\partial\Lambda^2, \\ \delta_\zeta M^{22} &= \bar{\zeta}^2\lambda^2 + i\bar{\zeta}^1\partial\Lambda^1, \\ \delta_\zeta \phi &= -\epsilon^{ij}(\bar{\zeta}^i\gamma_5\lambda^j + i\bar{\zeta}^i\gamma_5\partial\Lambda^j), \\ \delta_\zeta v_a &= -\epsilon^{ij}(i\bar{\zeta}^i\gamma_a\lambda^j + \bar{\zeta}^i\partial\gamma_a\Lambda^j), \\ \delta_\zeta \Lambda^i &= M^{ij}\bar{\zeta}^j - M^{jj}\bar{\zeta}^i + \frac{1}{2}\epsilon^{ij}\phi\gamma_5\zeta^j - \frac{i}{2}\epsilon^{ij}v_a\gamma^a\zeta^j - i\partial C\zeta^i, \\ \delta_\zeta C &= \bar{\zeta}^i\Lambda^i. \end{aligned} \quad (13)$$

where a general auxiliary field  $M^{12}$  appears in  $\delta_\zeta \lambda^i$  and  $\delta_\zeta \Lambda^i$  and its LSUSY transformation is given as

$$\delta_\zeta M^{12} = \bar{\zeta}^{(1}\lambda^{2)} - i\bar{\zeta}^{(1}\partial\Lambda^{2)}. \quad (14)$$

By means of the linearization of  $N = 2$  NLSUSY in  $d = 2$  [11], the LSUSY transformations (13) and (14) are obtained from the following composite expressions of the basic fields (except for constant shifts),

$$\begin{aligned} \lambda^i(\psi) &= \xi\psi^i|w|, \\ D(\psi) &= \frac{\xi}{\kappa}|w|, \\ M^{ij}(\psi) &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^j|w|, \\ \phi(\psi) &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|, \\ v_a(\psi) &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j|w|. \end{aligned}$$

$$\begin{aligned}
\Lambda^i(\psi) &= -\frac{1}{2}\xi\kappa^2\psi^i\bar{\psi}^j\psi^j|w|, \\
C(\psi) &= -\frac{1}{8}\xi\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j|w|,
\end{aligned} \tag{15}$$

under the  $N = 2$  NLSUSY transformations. We have also shown [11] the relation between the LSUSY action of Eq.(11) and the NLSUSY one of Eq.(1) for  $N = 2$  SUSY in  $d = 2$ , namely, the relation  $S_{N=2 \text{ gauge}} + [\text{surface term}] = \xi^2 S_{N=2 \text{ NLSUSY}}$ .

As for the above NL/LSUSY relation, we have found [13] that an ansatz  $\lambda^i(\psi) = \psi^i|w|$  in extended SUSY, which is the leading order of supercharges  $Q^i$  of SUSY, gives general LSUSY transformations for vector supermultiplets from functionals corresponding to Eq.(15), which are determined through Eqs.(8), (9) and (10). The  $N = 2$  LSUSY transformations (13) and (14) are derived from general LSUSY transformations in extended SUSY. Furthermore, the commutator-based linearization from Eqs.(8), (9) and (10) is practical in order to construct LSUSY supermultiplets in extended SUSY.

In curved spacetime, an Einstein-Hilbert-type NLSUSY invariant action is constructed in NLSUSY GR [9, 10] as follows;

$$\mathcal{S}_{\text{NLSUSYGR}} = -\frac{c^4}{16\pi G} \int d^4x |w|(\Omega + \Lambda), \tag{16}$$

In the action (16),  $|w| = \det w^a{}_\mu$  and  $w^a{}_\mu$  is defined as a unified vierbein  $w^a{}_\mu = e^a{}_\mu + t^a{}_\mu$ , where  $e^a{}_\mu$  is the vierbein in GR and  $t^a{}_\mu = -i\kappa^2\bar{\psi}^i\gamma^a\partial_\mu\psi^i$ . The  $\Omega$  means a scalar curvature in terms of  $(w^a{}_\mu, w^\mu{}_a)$  and  $\Lambda$  is a cosmological constant. In flat spacetime ( $e^a{}_\mu \rightarrow \delta^a{}_\mu$ ), the action (16) reduces to the VA NLSUSY action (1) with the dimensional constant  $\kappa$  being fixed to  $\kappa^{-2} = \frac{c^4\Lambda}{8\pi G}$ .

The NLSUSY-GR action (16) is invariant under NLSUSY transformations of  $\psi^i$  and  $e^a{}_\mu$

$$\begin{aligned}
\delta_\zeta\psi^i &= \frac{1}{\kappa}\zeta^i - i\kappa\bar{\zeta}^j\gamma^\mu\psi^j\partial_\mu\psi^i, \\
\delta_\zeta e^a{}_\mu &= 2i\kappa\bar{\zeta}^i\gamma^\nu\psi^i\partial_{[\mu}e^a{}_{\nu]},
\end{aligned} \tag{17}$$

which induce  $GL(4, \mathbf{R})$  transformations of  $w^a{}_\mu$ ,

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + w^a{}_\nu\partial_\mu\xi^\nu, \tag{18}$$

with  $\xi^\mu = -i\kappa\bar{\zeta}^i\gamma^\mu\psi^i$ . The NLSUSY transformations (17) satisfy the following commutator algebra,

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_{GL(4, \mathbf{R})}(\Xi^\mu), \quad (19)$$

where  $\delta_{GL(4, \mathbf{R})}(\Xi^\mu)$  is a  $GL(4, \mathbf{R})$  transformation with parameters

$$\Xi^\mu = 2(i\bar{\zeta}_1^i\gamma^\mu\zeta_2^i - \xi_1^\nu\xi_2^\rho e_a^\mu\partial_{[\nu}e_{\rho]}^a). \quad (20)$$

The NLSUSY-GR action (16) possesses large symmetries accomodating  $SO(N)$  SP group, as follows;

$$\begin{aligned} & [\text{global NLSUSY}] \otimes [\text{local } GL(4, \mathbf{R})] \otimes [\text{local Lorentz}] \\ & \otimes [\text{local spinor translation}] \otimes [\text{global } SO(N)] \otimes [\text{local } U(1)^N] \otimes [\text{chiral}]. \end{aligned} \quad (21)$$

Similar to the argument in flat spacetime, we discuss below possible functionals of  $\psi^i$  and  $e^a_\mu$  under the commutator algebra (19). For simplicity, let us first consider Lorentz- and Riemann-tensor functionals of  $\psi^i$  and  $e^a_\mu$  without their derivative terms,

$$f^{IA}_M = f^{IA}_M(\psi^i, e^a_\mu), \quad g^{JB}_N = g^{JB}_N(\psi^i, e^a_\mu), \quad (22)$$

with Riemann spacetime indices  $M, N = \mu, \mu\nu, \dots$ , which satisfy the NLSUSY algebra (19), i.e.

$$\begin{aligned} [\delta_{\zeta_1}, \delta_{\zeta_2}]f^{IA}_M &= \Xi^\rho\partial_\rho f^{IA}_M + \sum_k f^{IA}_{M_k(\rho)}\partial_{\mu_k}\Xi^\rho, \\ [\delta_{\zeta_1}, \delta_{\zeta_2}]g^{JB}_N &= \Xi^\rho\partial_\rho g^{JB}_N + \sum_k g^{JB}_{N_k(\rho)}\partial_{\mu_k}\Xi^\rho. \end{aligned} \quad (23)$$

In Eq.(23), Riemann spacetime indices  $M_k(\rho)$  are defiend as  $M_k(\rho) = \mu_1\mu_2\cdots\rho\cdots\mu_n$  and the summation  $\sum_k$  means

$$\begin{aligned} \sum_k f^{IA}_{M_k(\rho)}\partial_{\mu_k}\Xi^\rho &= f^{IA}_{M_1(\rho)}\partial_{\mu_1}\Xi^\rho + f^{IA}_{M_2(\rho)}\partial_{\mu_2}\Xi^\rho + \cdots \\ &= f^{IA}_{\rho\mu_2\cdots}\partial_{\mu_1}\Xi^\rho + f^{IA}_{\mu_1\rho\cdots}\partial_{\mu_2}\Xi^\rho + \cdots. \end{aligned} \quad (24)$$

Then, the commutator algebra for the products  $f^{IA}_M g^{JB}_N$  are also closed as

$$\begin{aligned} [\delta_{\zeta_1}, \delta_{\zeta_2}](f^{IA}_M g^{JB}_N) &= \Xi^\rho\partial_\rho(f^{IA}_M g^{JB}_N) + \sum_k f^{IA}_{M_k(\rho)}\partial_{\mu_k}\Xi^\rho g^{JB}_N + f^{IA}_M \sum_k g^{JB}_{N_k(\rho)}\partial_{\mu_k}\Xi^\rho \\ &= \delta_{GL(4, \mathbf{R})}(f^{IA}_M g^{JB}_N)(\Xi^\rho). \end{aligned} \quad (25)$$

Since the NLSUSY transformations (17) satisfy the algebra (19), all Lorentz- and Riemann-tensor functionals expressed by means of (22) satisfy the algebra (19).

Next we consider functionals including derivatives of  $\psi^i$  and  $e^a_\mu$ . As for commutator algebras for their derivative terms under the NLSUSY transformations (17), only for  $\partial_\mu \psi^i$  and  $\partial_{[\mu} e^a_{\nu]}$  are closed as

$$\begin{aligned} [\delta_{\zeta_1}, \delta_{\zeta_2}] \partial_\mu \psi^i &= \Xi^\nu \partial_\nu (\partial_\mu \psi^i) + (\partial_\nu \psi^i) \partial_\mu \Xi^\nu = \delta_{GL(4, \mathbf{R})}(\partial_\mu \psi^i)(\Xi^\nu), \\ [\delta_{\zeta_1}, \delta_{\zeta_2}] \partial_{[\mu} e^a_{\nu]} &= \Xi^\rho \partial_\rho (\partial_{[\mu} e^a_{\nu]}) + (\partial_{[\rho} e^a_{\nu]}) \partial_\mu \Xi^\rho + (\partial_{[\mu} e^a_{\rho]}) \partial_\nu \Xi^\rho \\ &= \delta_{GL(4, \mathbf{R})}(\partial_{[\mu} e^a_{\nu]})(\Xi^\nu). \end{aligned} \quad (26)$$

The commutator algebra for the higher-order derivative terms of  $\psi^i$  and  $e^a_\mu$  (i.e. for  $(\partial^2 \psi^i, \partial^2 e^a_\mu, \dots)$ ) are not closed. Therefore, in curved spacetime the following Lorentz- and Riemann-tensor functionals expressed only in terms of  $\psi^i$ ,  $e^a_\mu$  and their first-order derivatives,

$$f^{IA}_M = f^{IA}_M(\psi^i, e^a_\mu; \partial_\mu \psi^i, \partial_{[\mu} e^a_{\nu]}) \quad (27)$$

satisfy the commutator algebra (19) from the same arguments as Eqs.(23) and (25).

Furthermore, in the same way as the commutator-based arguments in flat spacetime, we expect that the two supertransformations for some functionals of  $\psi^i$  and  $e^a_\mu$  determine general (global) LSUSY transformations for linear supermultiplets in curved spacetime, which are caused by the closure of the commutator algebra (19).

We summarize our results as follows. In this letter, under the NLSUSY transformations (3) and (17) we have shown the closure of the commutator algebra for the Lorentz-tensor functionals (5) in flat spacetime and for the Lorentz- and Riemann-tensor functionals (27) in curved spacetime. Based on those arguments, we have pointed out that the commutator-based LSUSY transformations (10) are (uniquely) determined from the two supertransformations (8) and the variations (9), which lead to linear supermultiplets with general auxiliary fields. The general LSUSY supermultiplets as shown in the example of Eqs.(13), (14) and (15) are constructed from examining the commutator algebra (6) for NLSUSY (and also Eq.(19) for NLSUSY GR) prior to transforming to gauge supermultiplets. In particular, the commutator-based linearization from Eqs.(8), (9) and (10) is practical in order to find general LSUSY supermultiplets in extended SUSY.



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